Proof – Year 1

The following table contains pairs of statements, A and B.

In each case complete the central column with one of the three symbols $\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, or with 'none'.

Also in each case, add the letters 'N' (necessary), 'S' (sufficient), or 'N & S' (necessary and sufficient), or 'neither', to indicate the relationship between the statements.

Q. no.	A nec. or suff. for B?	Statement A	$\Rightarrow$, $\Leftarrow$, $\Leftrightarrow$, 'none'?	Statement B	B nec. or suff. for A?
1		$x^2 = 9$		$x < 6$	
2		This month has exactly 29 days		It is February in a leap year	
3		$x = +1$ or $x = -1$		$(x+1)(x-1) = 0$	
4		$\sin x = 1$		$x = 90°$	
5		A polygon is a square		A polygon has 4 sides	
6		$(x+1)(y-1) = 0$		$x = -1$ and $y = +1$	
7		Three straight lines in 2d meet in exactly 2 points		Exactly 2 of 3 straight lines in 2d are parallel	
8		$x^2 = 9$		$x = 3$	
9		n^2 is a multiple of 5		n is a multiple of 5	
10		k is divisible by 3		$k + 1$ is even	
11		The discriminant of a quadratic equation is non-negative		A quadratic equation has 2 unequal roots	
12		$a > b$		$\frac{1}{a} < \frac{1}{b}$	

Exercise 1.1

Q1 Which of the following statements is true?
A: $x^2 > 9 \Rightarrow x > 3$ B: $x^2 > 9 \Leftarrow x > 3$ C: $x^2 > 9 \Leftrightarrow x > 3$

Q2 a) Prove that the sum of two odd numbers is even.
b) Prove that the product of two even numbers is even.
c) Prove that the product of an odd number and an even number is even.

Q3 By finding a counter-example, disprove the following statement:
"If p is a non-zero integer, then $\frac{1}{p^2} < \frac{1}{p}$."

Q4 Prove that, for any integer x, $(x + 5)^2 + 3(x - 1)^2$ is always divisible by four.

Q5 Prove by exhaustion that the product of any three consecutive integers is even.

Q6 Disprove the following statement:
"$n^2 - n - 1$ is a prime number for any integer $n > 2$."

Q7 Disprove the following: $\sqrt{x^2 + y^2} < x + y$.

Q8 Prove that the sum of two rational numbers is also a rational number.

Q9 a) Prove the statement below:
"For any integer n, $n^2 - n - 1$ is always odd."
b) Hence prove that $(n^2 - n - 2)^3$ is always even.